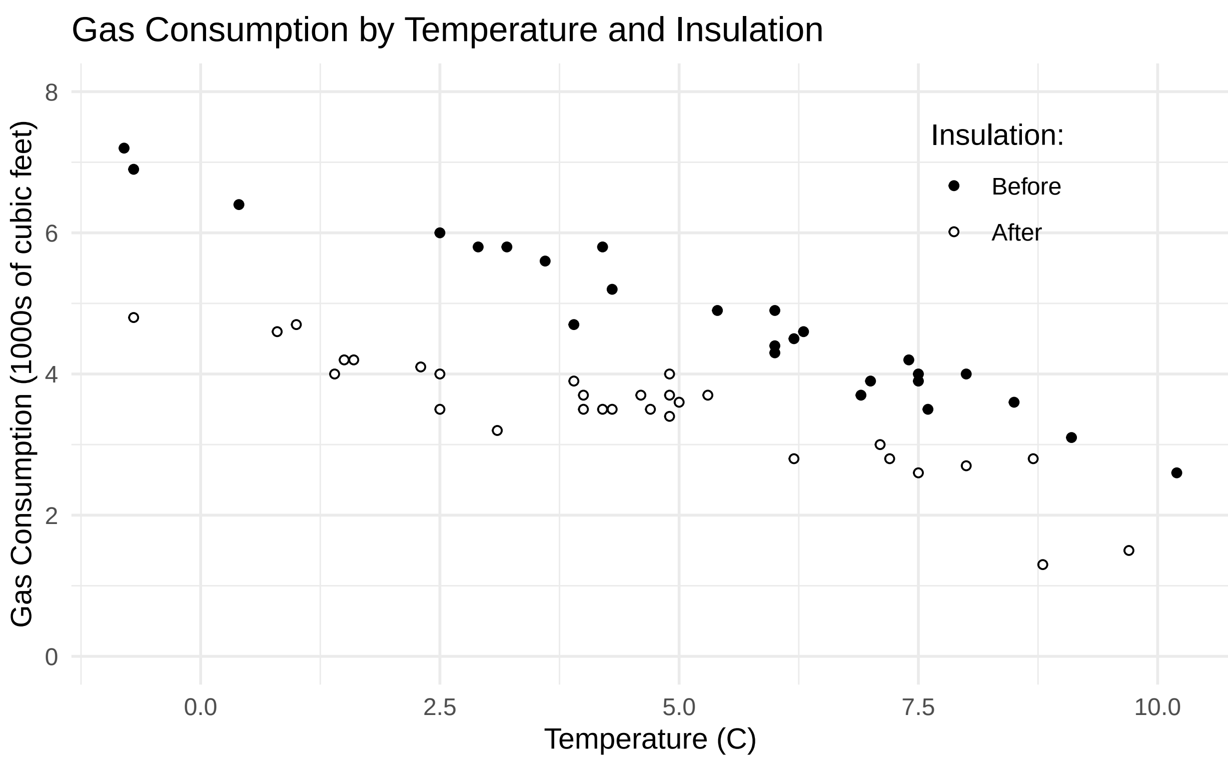
Wednesday, January 8

Motivating Examples

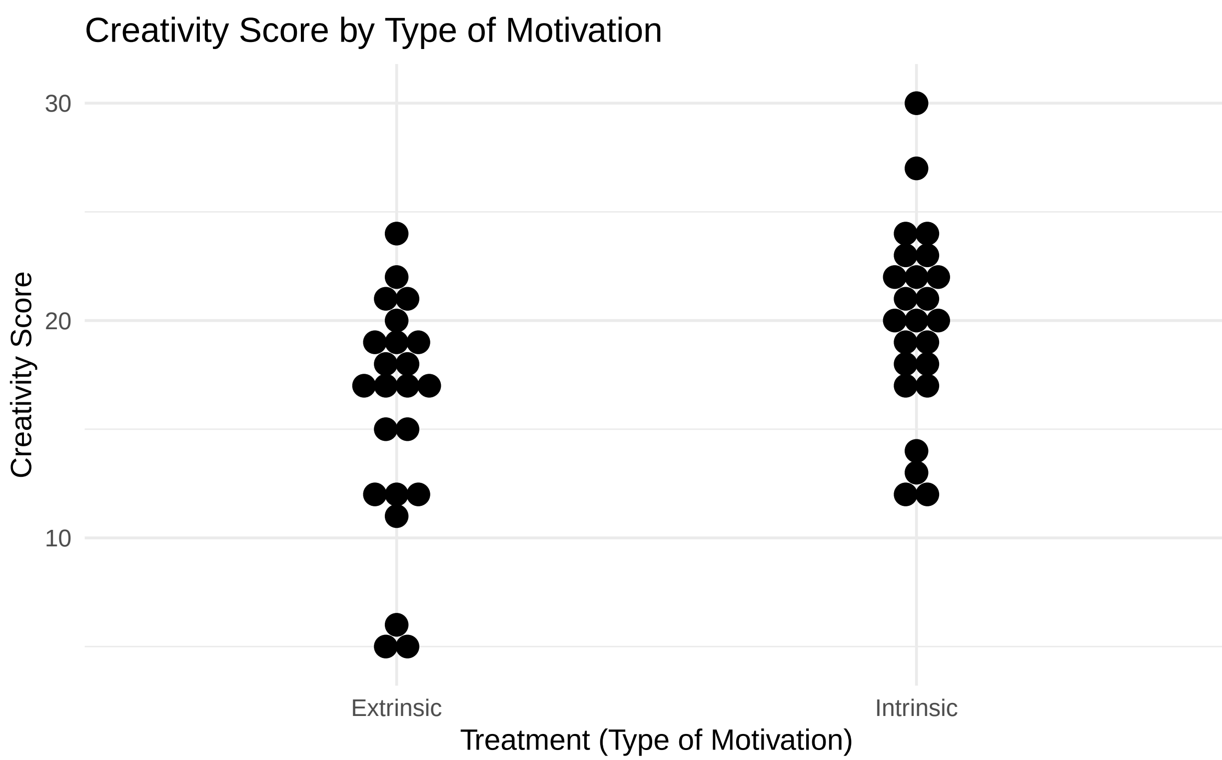
Example: The data below show observations of the average outdoor temperature (C) and gas consumption (in 1000s of cubic feet) to heat a home for a week. But note that between the 26th and 28th weeks cavity-wall insulation was added.

Week	Gas	Insulation	Temp
1	3.7	Before	6.9
2	3.9	Before	7.5
3	4	Before	7.5
⋮	⋮	⋮	⋮
26	4	Before	8
28	4.2	After	1.6
29	1.3	After	8.8
30	3.9	After	3.9
⋮	⋮	⋮	⋮
57	2.7	After	8



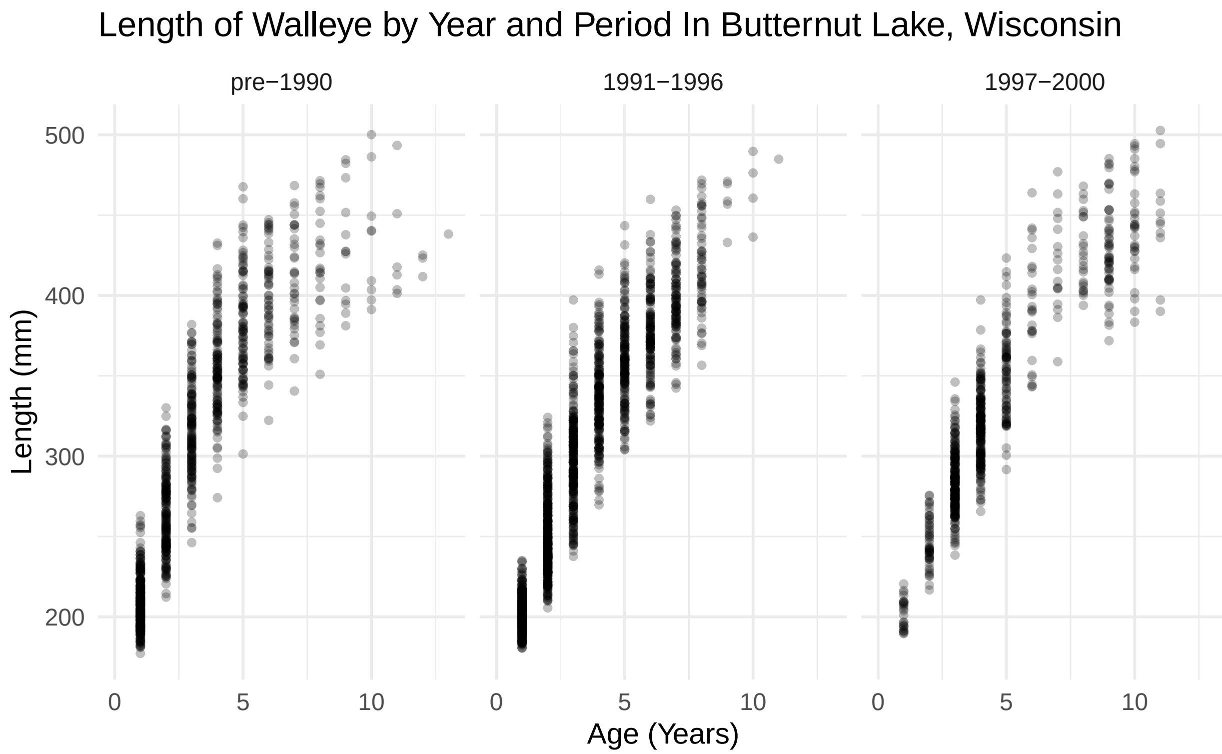
Example: Creative writing students were treated or "primed" with either extrinsic or intrinsic motivation. They were then asked to write a poem in the Haiku style about laughter. Each poem was then scored for "creativity" on a 40-point scale by 12 judges. These scores were then averaged across the 12 judges for each student.

Score	Treatment
5	Extrinsic
5	Extrinsic
6	Extrinsic
⋮	⋮
24	Extrinsic
12	Intrinsic
12	Intrinsic
13	Intrinsic
⋮	⋮
30	Intrinsic



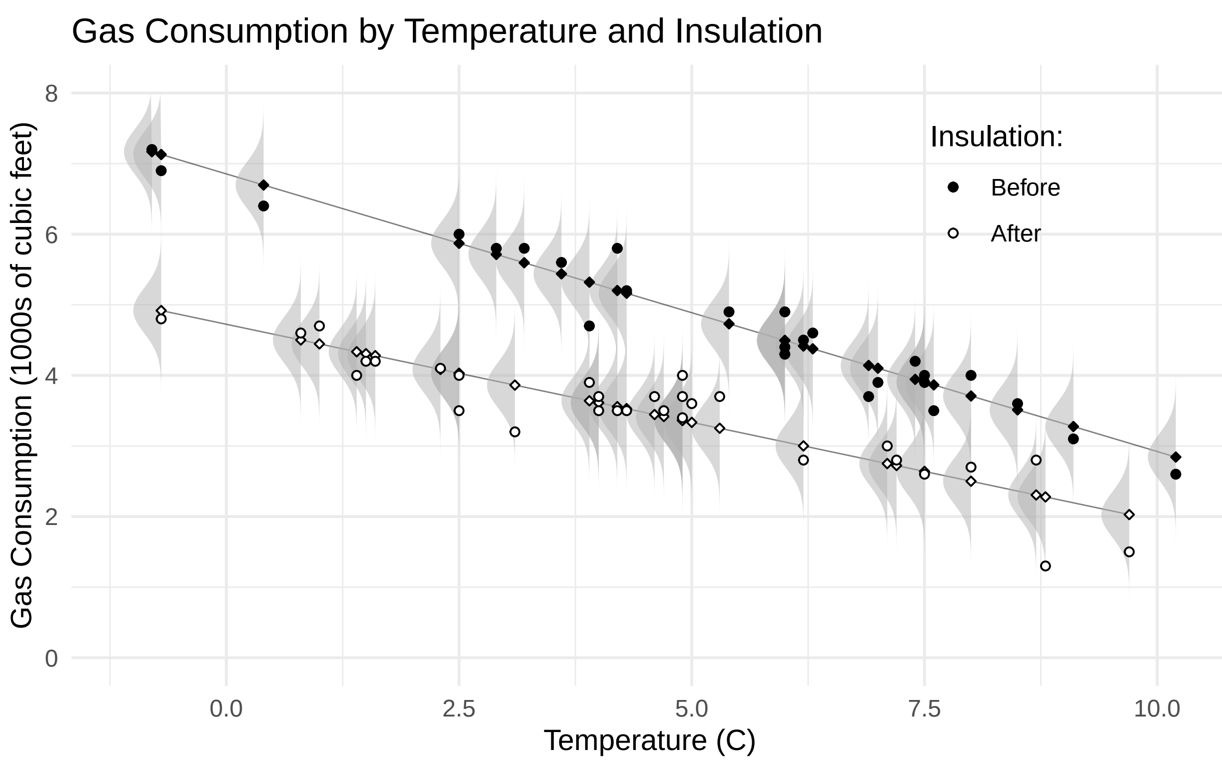
Example: These are data on 3198 walleye captured in Butternut Lake, Wisconsin, during three periods with different management methods in place.

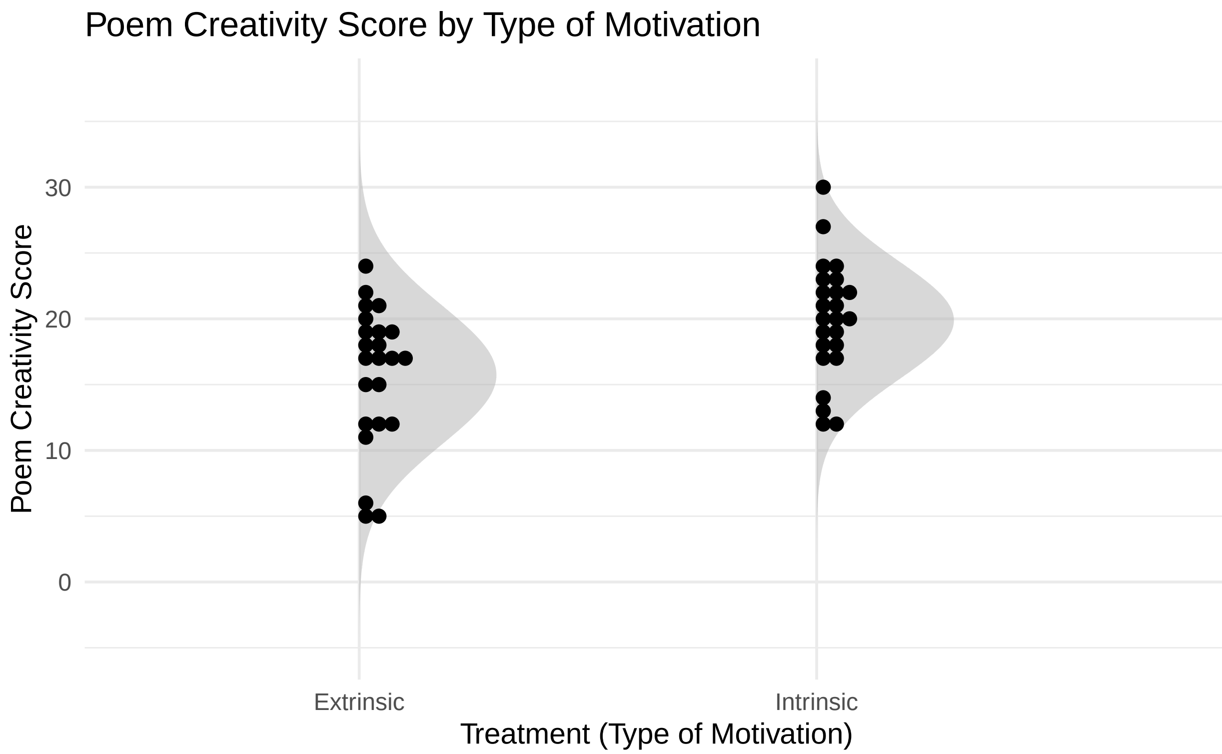
Length	Age	Period
215	1	pre-1990
193	1	pre-1990
203	1	pre-1990
201	1	pre-1990
232	1	pre-1990
⋮	⋮	⋮
397	11	1997-2000



A Statistical Model for the Response Variable

We might consider a model for the *distribution* of the *response* variable, where one or more properties of the *distribution* of Y are *functions of* the *explanatory* variable(s).





One common property of the distribution of a response variable is the *mean* or *expected value* of the variable.

The Data Structure

We let Y_i denote the i-th observation of a *response* variable, and X_{ij} denote the i-th observation of the j-th *explanatory* variable. Assume n observations ($i = 1, 2, \ldots, n$) and k explanatory variables ($j = 1, 2, \ldots, k$).

$$\begin{array}{cc} Y_1 & X_{11}, X_{12}, \ldots, X_{1k} \\ Y_2 & X_{21}, X_{22}, \ldots, X_{2k} \\ \vdots & \vdots \\ Y_n & X_{n1}, X_{n2}, \ldots, X_{nk} \end{array}$$

Sometimes when it is not necessary to refer to a specific observation, we will omit the i subscript to denote the response variable and explanatory variables as follows.

$$Y \quad X_1, X_2, \ldots, X_k$$

How do we usefully *model* the *statistical* relationship between the response variable (i.e., Y) and one or more explanatory variables (i.e., $X_1, X_2, \ldots, X_k$)?

Expectation

The **expected value** of a random variable Y is defined as

$$E(Y) = \sum_y yP(Y = y)$$

if it is *discrete*, and

$$E(Y) = \int_{-\infty}^{\infty} yf(y)dy$$

if it is *continuous*. Note that by convention, an upper-case letter (e.g., Y) denotes a random variable whereas a lower-case letter (e.g., y) denotes a *value* or *realization* of that random variable.

Models for $E(Y)$

Most regression models focus on the mathematical relationship between the *expectation of the response variable* to the *value(s) of the explanatory variable(s)*. That is, $E(Y)$ is a *function of* $x_1, x_2, \dots, x_k$.

One way to write a regression model is

$$E(Y) = f(x_1, x_2, \dots, x_k),$$

where f is some specified function. For example,

$$E(Y) = \beta_0 + \beta_1 x,$$

where the subscript is dropped without loss of clarity from x_1 as there is only one explanatory variable. With two or more explanatory variables we might have

$$E(Y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_k x_k.$$

These are examples of *linear models* (more on that soon). We will also consider *nonlinear* models like

$$E(Y) = \alpha + (\delta - \alpha)e^{-x \log(2)/\gamma}.$$

Typically the function will involve constants (e.g., $\beta_0, \beta_1, \dots, \beta_k$ or α, δ, and γ) that are unknown, but often of interest. These are the **parameters** of the regression model.