Wednesday, December 3

Bias Due to Non-response

Assume two strata: N_r responders, and N_m non-responders (m for *missing*). Thus

$$\mu = \frac{N_r}{N}\mu_r + \frac{N_m}{N}\mu_m,$$

where μ_r and μ_m are the mean responses of all responders and all non-responders, respectively.

Assume a simple random sampling design. The sample is only from the stratum of *responders*. We compute $\bar{y}_r$ (i.e., the mean response of the sample of responders) to estimate μ (the mean response of all responders and non-responders). The bias of $\bar{y}_r$ with respect to μ is

$$E(\bar{y}_r) - \mu = \mu_r - \left(\frac{N_r}{N}\mu_r + \frac{N_m}{N}\mu_m\right) = \frac{N_r + N_m}{N}\mu_r - \frac{N_r}{N}\mu_r - \frac{N_m}{N}\mu_m = \frac{N_m}{N}(\mu_r - \mu_m),$$

noting that $N = N_r + N_m$. When is the bias *large* in absolute value?

Methods for Correcting for Non-Response Bias

1. Double sampling.
2. Stratification.
3. Calibration.
4. Re-weighting.
5. Imputation.
6. Incentivize.

Double Sampling for Non-Response

The double sampling method is effectively a variation on the use of double sampling in stratified random sampling.

1. Obtain a simple random sample of n' respondents. Attempt to elicit responses from all n' respondents. This sample will include n'_r responders and n'_m non-responders (i.e., $n' = n'_r + n'_m$).

2. Obtain responses from a sample of n_m non-responders. This is a sample of the original n'_m non-responders. Note that we do not need to sample from the responders as we already have responses from them (i.e., $n_r = n'_r$). The estimator of μ is then

   $$\hat{\mu} = \frac{n'_r}{n'}\bar{y}_r + \frac{n'_m}{n'}\bar{y}_m,$$

   where $\bar{y}_r$ and $\bar{y}_m$ are the sample means for the samples of n_r responders and n_m non-responders, respectively.

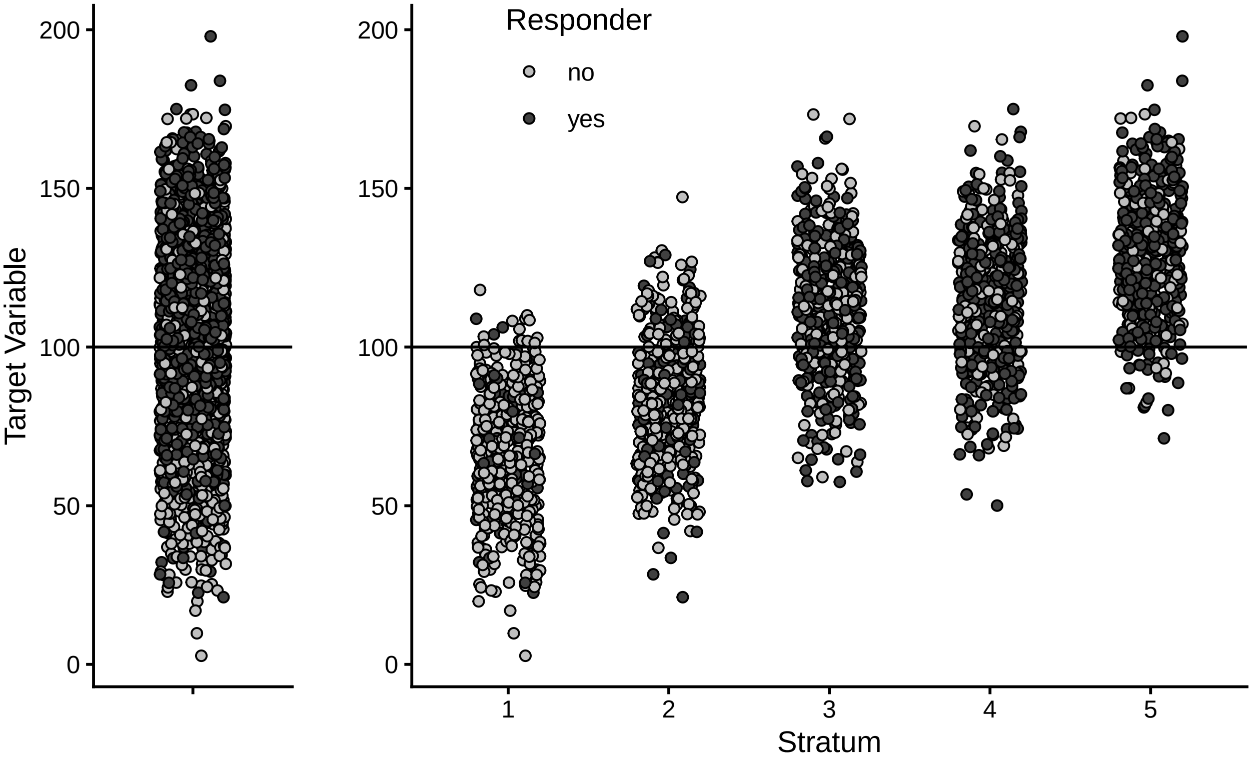
Note: This assumes that the sample of n_m non-responders that ultimately respond are representative of the stratum of non-responders (i.e., $\bar{y}_m$ is an unbiased estimator of μ_m).

Example: In a simple random sample of 120 people, 30 people respond and the other 90 do not. Of the 30 people who responded, suppose that the mean response is $\bar{y}_r = 5$. Suppose that of the 90 non-responders, we are able to obtain responses from 25 respondents, yielding a mean response of $\bar{y}_m = 2$. Assuming that $\bar{y}_m$ is an unbiased estimator of μ_m, what is our estimate of μ?

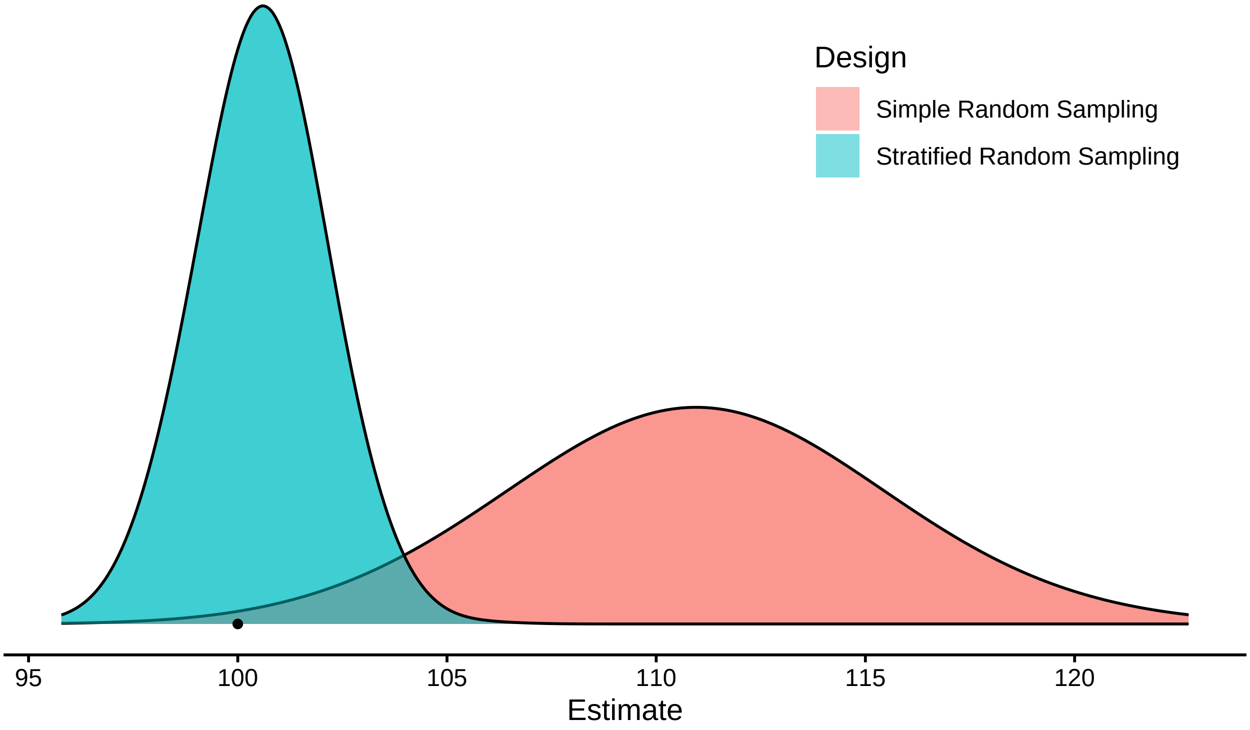
Stratification by Response Rate/Probability

Suppose we can also stratify respondents into strata such that for respondents in the same stratum there is little or no relationship between the target variable and responding.

Example: Suppose we have a population where responders tend to have larger values of the target variable, but we can stratify the population such that the relationship between responding and the target variable disappears *within* each stratum.



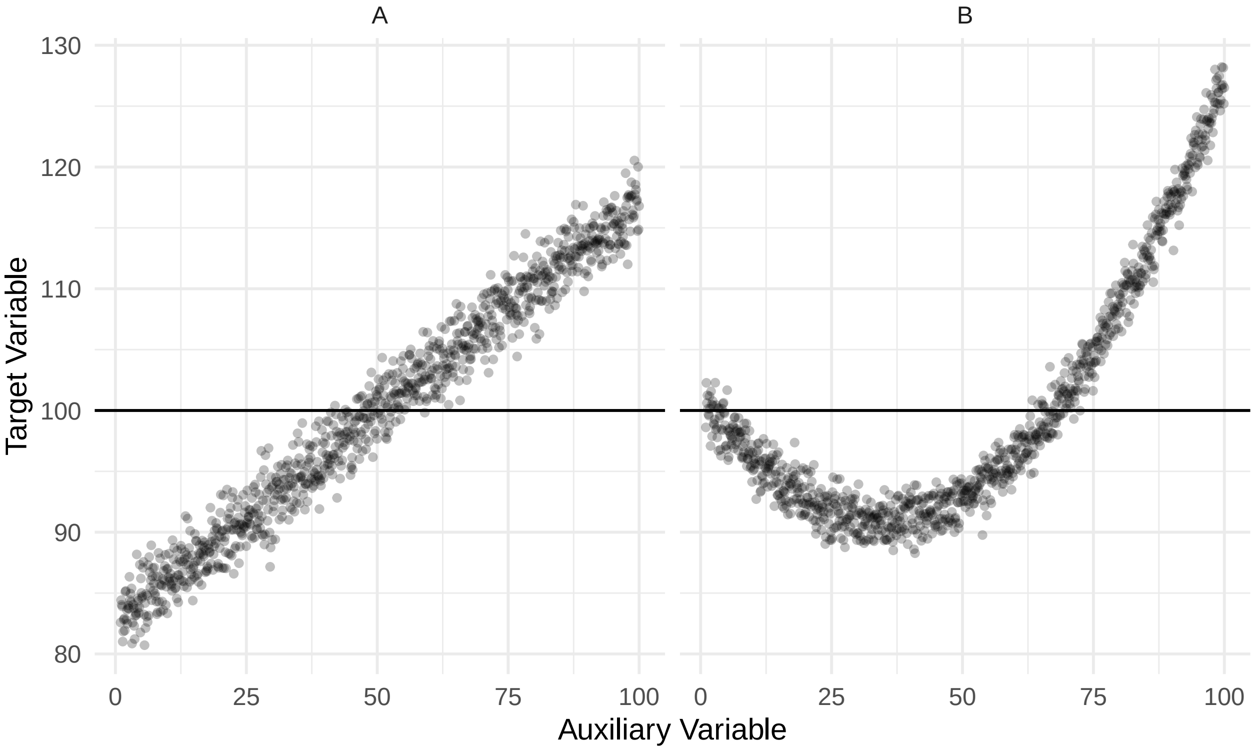
Now consider two designs to estimate μ: *simple random sampling* and *stratified random sampling*. We have that $\mu = 100$. How do these designs perform?



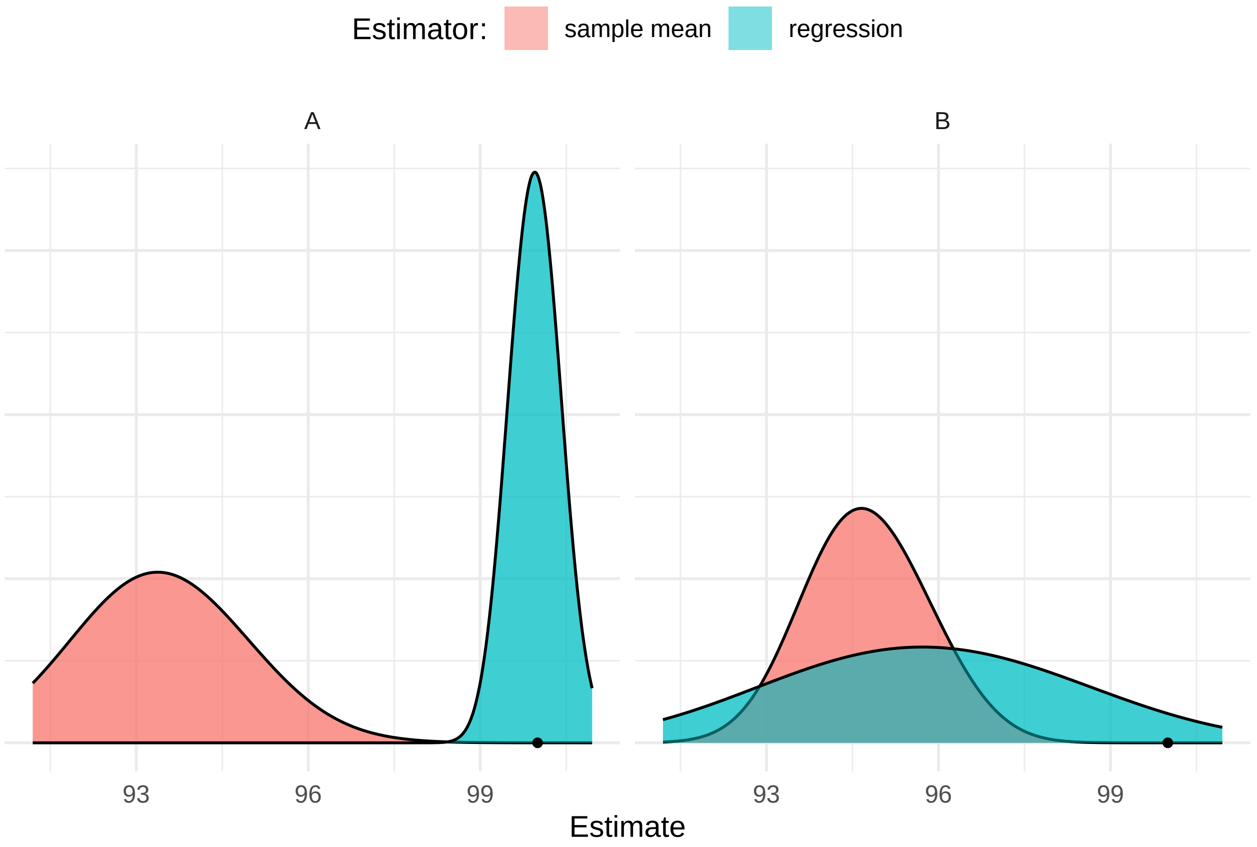
Calibration

Calibration and its various special cases (e.g., ratio and regression estimators, post-stratification) can compensate for non-response *provided* that the assumed statistical relationship between the target variable and auxiliary variable(s) is accurate.

Example: The following plot shows two populations. In both populations $\mu = 100$ and the probability of non-response increases with the value of the auxiliary variable.



Assume a simple random sampling design and two estimators: the *sample mean* and a *regression estimator*. Recall that for both populations $\mu = 100$.



Re-Weighting

Recall that we can estimate τ as

$$\hat{\tau} = \sum_{i \in \mathcal{S}} \frac{y_i}{\pi_i},$$

where π_i is the probability of inclusion of the i-th element in the sample $\mathcal{S}$. To be included the element must be (a) *sampled* by the sampling process and (b) *respond*. Thus

$$\pi_i = P(\text{the } i\text{-th unit is sampled})P(\text{the } i\text{-th unit responds} \mid \text{the } i\text{-th unit is sampled}).$$

It is useful to write the inclusion probability as

$$\pi_i = \underbrace{P(\text{the } i\text{-th unit is sampled})}_{\pi_{i,S}}\underbrace{P(\text{the } i\text{-th unit responds} \mid \text{the } i\text{-th unit is sampled})}_{\pi_{i,R}},$$

so that $\pi_i = \pi_{i,S}\pi_{i,R}$. Here $\pi_{i,R}$ is the probability that the i-th unit will respond (if sampled). Also recall that we can turn $\hat{\tau}$ into an estimate of μ by dividing $\hat{\tau}$ by (an estimate of) the number of elements in the population.

If we *know* $\pi_{i,R}$ we could effectively adjust for non-response. But in practice we need to *estimate* it. There are a variety of ways of doing this.

1. **Weight-class adjustment**. Suppose we can post-stratify the sample of *sampled* elements where we assume the response rate is approximately the same within strata. Then we can estimate $\pi_{i,R}$ as *the proportion of responders within each stratum.*
2. **Model-based adjustment**. If we are willing to assume that $\pi_{i,R}$ is a function of one or more auxiliary variables, we could estimate a model (e.g., logistic regression) that allows us to estimate the probability of a response from a respondent given the value(s) of their auxiliary variable(s).

Imputation

Imputation involves *predicting the response* of non-responders. There are a couple of issues that should be considered when using imputation.

1. The method of imputing missing responses should not tend to overestimate or underestimate those responses, otherwise some (or more) bias will remain.
2. Uncertainty in the imputed responses should be taken into account in inferences (i.e., variances of estimators and thus bounds on the error of estimation). One powerful method of doing this is *multiple imputation.*

Imputation is also very useful for “filling-in” missing values of auxiliary variables (e.g., age, gender).

Incentivize Responding

Where possible, incentivize respondents to respond.

1. Encourage responding (e.g., follow-up initial invitations, stress importance of participation).
2. Discourage avoidance (e.g., take measures to assure anonymity or confidentiality).