Monday, Nov 11

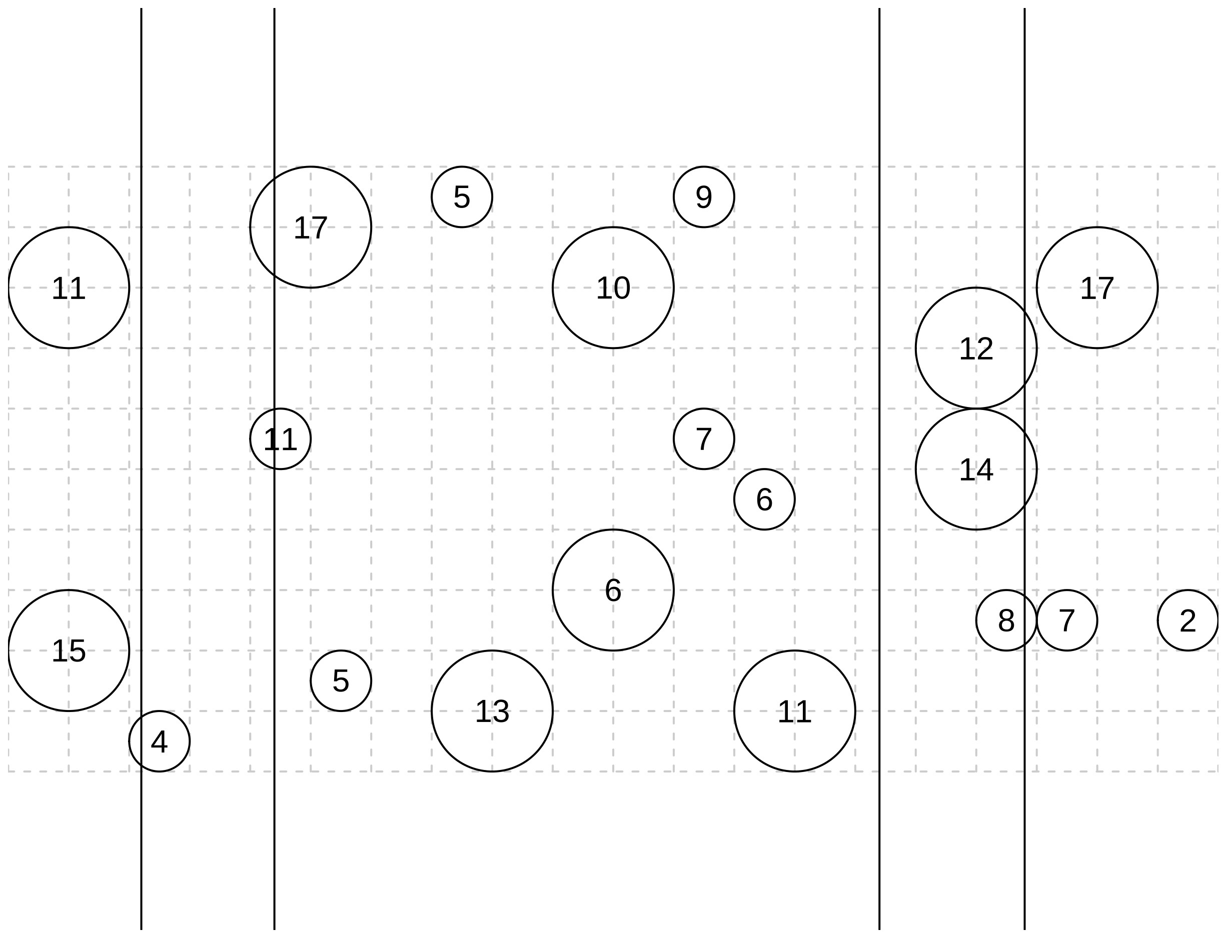
Line-Intercept Sampling

A sample of elements in a region are selected according to the following procedure.

1. Select a random point within the region based on a uniform distribution. Extend a transect line from that point in a given direction such that it crosses the whole region.
2. Select all objects that are intercepted by the transect line.

This process can be repeated.

Example: Consider the line-intercept survey with four transect lines.



Consider the sample of objects intercept by *one line*. The inclusion probability of the i-th object is $\pi_i = w_i/W$, where w_i is the horizontal width of the object and W is the total width of the region. An estimator of τ based on a given line is the Horvitz-Thompson estimator

$$\hat{\tau} = \sum_{i \in \mathcal{S}} \frac{y_i}{\pi_i} = W \sum_{i \in \mathcal{S}} \frac{y_i}{w_i}.$$

Example: Based on the example shown above, what are the estimates of τ based on the three lines that intercept objects? The value of the target variable is shown within each object.

Let $\hat{\tau}_k$ be the estimate of τ based on the k-th non-empty line. An estimate of τ can be obtained by averaging K transect line estimates to get

$$\hat{\tau} = \frac{1}{K} \sum_{k=1}^{K} \hat{\tau}_k.$$

The estimated variance of $\hat{\tau}$ is then

$$\hat{V}(\hat{\tau}) = \frac{1}{K(K-1)} \sum_{k=1}^{K} (\hat{\tau}_k - \hat{\tau})^2.$$

Example: What is the estimate of τ based on the survey given earlier?

Another estimator is to use a Horvitz-Thompson estimator based on the sample of elements intersected by the n transect lines. Then the Horvitz-Thompson estimator is

$$\hat{\tau} = \sum_{i \in \mathcal{S}} \frac{y_i}{\pi_i},$$

where $\pi_i = 1 - (1 - w_i/W)^t$, since we are sampling with replacement and w_i/W is the selection probability of the i-th element. Calculation of the estimated variance of this estimator requires the second-order (joint) inclusion probabilities, which can be computed as

$$\pi_{ij} = \pi_i + \pi_j - 1 + \left(1 - \frac{w_i + w_j - w_{ij}}{W}\right)^t,$$

where w_{ij}/W is the probability that objects i and j would both be intersected by a line.

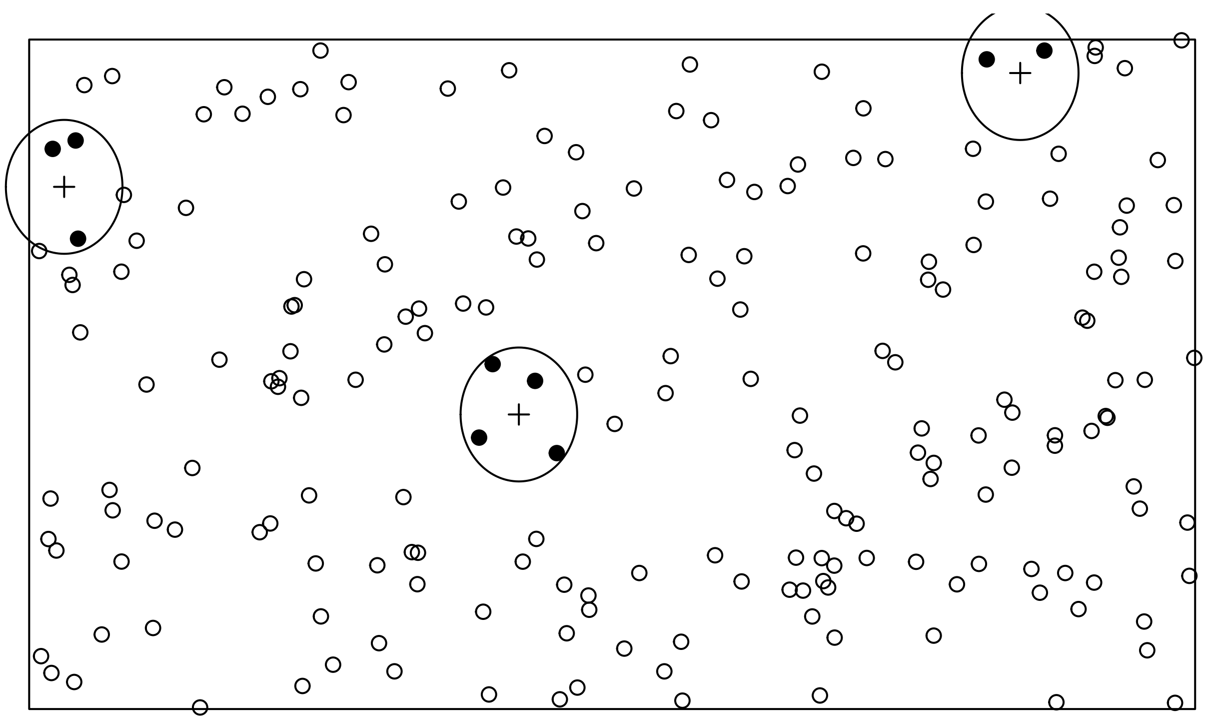
Fixed Area Plot Sampling

A sample of objects in a region are selected according to the following procedure.

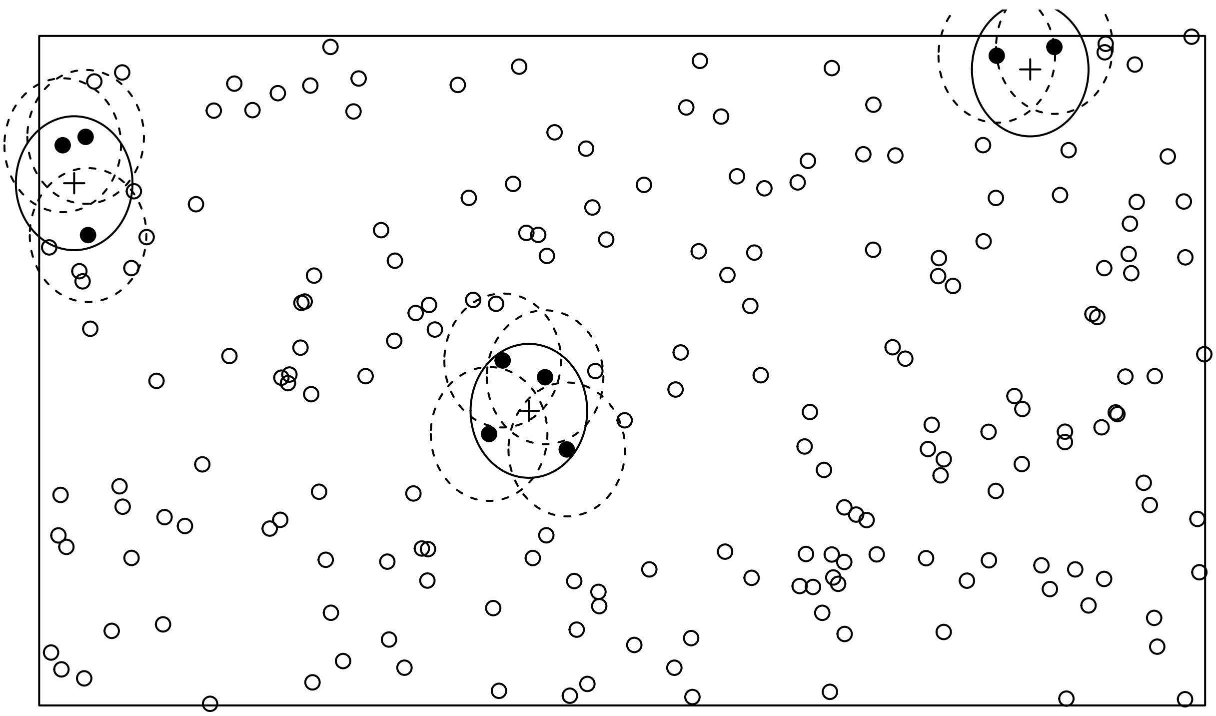
1. Select a random point within the region based on a uniform distribution.
2. Select all objects that are within a plot of a given shape (e.g., circle, square, or rectangle) centered on that point.

This process can be repeated.

Example: Consider the following fixed area plot survey with three circular fixed area plots.



The probability that an object will be included within a plot equals the area of the plot that is also within the region when the center of the plot is centered on the object.



For a given plot the Horvitz-Thompson estimator of τ is

$$\sum_{i \in \mathcal{S}} \frac{y_i}{\pi_i} = A \sum_{i \in \mathcal{S}} \frac{y_i}{a_i},$$

because $\pi_i = a_i/A$ where A is the total area of the region and a_i is the area of the plot that is also withiin the region when the plot is centered on the i-th object.

An estimator of τ can be obtained by averaging these estimates. Let $\hat{\tau}_k$ be the estimate from the k-th non-empty plot. If there are K non-empty plots then the estimator of τ is

$$\hat{\tau} = \frac{1}{K} \sum_{k=1}^{K} \hat{\tau}_k.$$

The estimated variance of $\hat{\tau}$ is

$$\hat{V}(\hat{\tau}) = \frac{1}{K(K-1)} \sum_{k=1}^{K} (\hat{\tau}_k - \hat{\tau})^2.$$

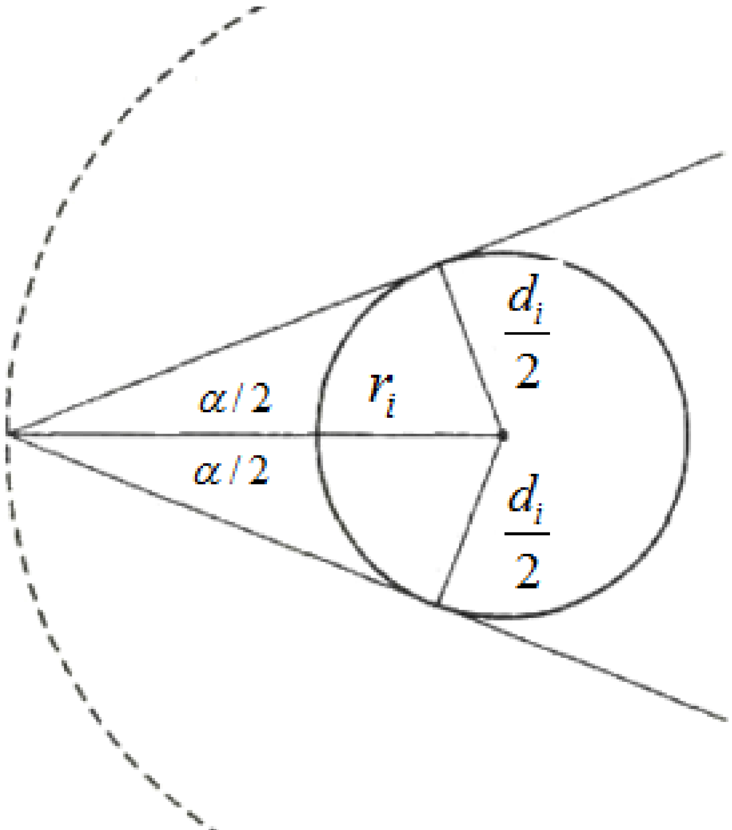
Example: Assume a region with a total area of 5000 meters and $K = 3$ non-empty circular plots.

k	y_i	a_i
1	63.9	59.5
1	50.4	74.3
1	61.8	75.7
2	27.1	78.5
2	42.0	78.5
2	52.5	78.5
2	33.4	78.5
3	27.8	53.2
3	57.8	47.7

Bitterlich Sampling

A sample of trees is selected according to the following procedure.

1. Select a random point within a region based on a uniform distribution.
2. Select all trees with trunk diameters that exceed a critical angle (α) when viewed from that point.



Also see Figure 2 in this paper.

Let α be the critical angle and d_i by the diameter of the i-th tree. Then the radius (r_i) of a circle that encloses all points that would result in the selection of the i-th tree is

$$r_i = \frac{d_i}{2\sin(\alpha/2)}.$$

Thus the i-th tree is selected if and only if

$$\text{distance to center of the } i\text{-th tree} \leq \frac{d_i}{2\sin(\alpha/2)},$$

assuming that this circle does not extend outside the region. The inclusion probability of the i-th tree is the probability of this happening. The area of this circle is $a_i = \pi r_i^2$ which can be computed as

$$a_i = \frac{\pi d_i^2}{4\sin^2(\alpha/2)}.$$

Note that π here is the mathematical constant $\pi \approx 3.14$, *not* the inclusion probability of the tree. The inclusion probability of the i-th tree is

$$\pi_i = a_i/A,$$

where A is the total area of the region in which the point was sampled. If there are n selected trees, then the estimate of τ for some target variable y_i is

$$\hat{\tau} = \sum_{i \in \mathcal{S}} \frac{y_i}{\pi_i}.$$

Variations on Bitterlich Sampling

1. If y_i is the *basal area* of the i-th tree so that $y_i = \pi d_i^2/4$, then

$$\hat{\tau} = \sum_{i \in \mathcal{S}} \frac{y_i}{\pi_i} = nA \sin^2(\alpha/2),$$

   where n is the number of selected trees, so that the estimated total basal area is proportional to the number of selected trees.

2. As in the previous examples if we have K estimates of τ (based on as many points) then these can be averaged to come up with one estimate. In the case of estimating total basal area, this estimator becomes

$$\hat{\tau} = \frac{A \sin^2(\alpha/2)}{K} \sum_{k=1}^{K} n_k,$$

   so that $\hat{\tau}$ is proportional to the average number of trees selected.