Monday, Mar 6

Statistical Significance

A **statistically significant** result is one that is decidedly not due to "ordinary variation" in the data (i.e., not due to chance or not a coincidence). **Statistical tests** (aka *significance tests* or *statistical hypothesis tests* or *hypothesis tests*) are how we decide whether or not an observed result is statistically significant.

Is a Coin Fair?

Suppose we flip a coin n times. We can consider the observation of each flip to be a random variable with the following distribution.

x	$P(x)$
Heads	p
Tails	$1 - p$

The value of p implies something about the coin.

1. If $p = 0.5$ the coin is fair.
2. If $p \neq 0.5$ the coin is not fair.

Assume we do not know the value of p. We flip the coin 30 times to produce a sample of $n = 30$ observations. It comes up heads 20 times, so $\hat{p} = 20/30 = 2/3 \approx 0.67$. What might we decide about p?

1. Conclude that $p = 0.5$. The result that $\hat{p} = 2/3$ **is not** statistically significant.
2. Conclude that $p \neq 0.5$. The result that $\hat{p} = 2/3$ **is** statistically significant.

How do we decide?

Can Milena Read?

Suppose Milena plays n games of Pounce. We can consider the observation of her response to a single game to be a random variable with the following distribution.

x	$P(x)$
Correct	p
Incorrect	$1 - p$

The value of p implies something about Milena's reading ability.

1. Milena cannot read. She is guessing so $p = 1/3$.
2. Milena can read (somewhat) so $p > 1/3$.

We do not know the value of p. Milena played Pounce 50 times to produce a sample of $n = 50$ observations. She selected the correct word 25 times, so $\hat{p} = 25/50 = 0.5$. What would we decide about p?

1. Conclude that $p = 1/3$. The result that $\hat{p} = 0.5$ **is not** statistically significant.
2. Conclude that $p > 1/3$. The result that $\hat{p} = 0.5$ **is** statistically significant.

What do we decide?

The Sampling Distribution of $\hat{p}$

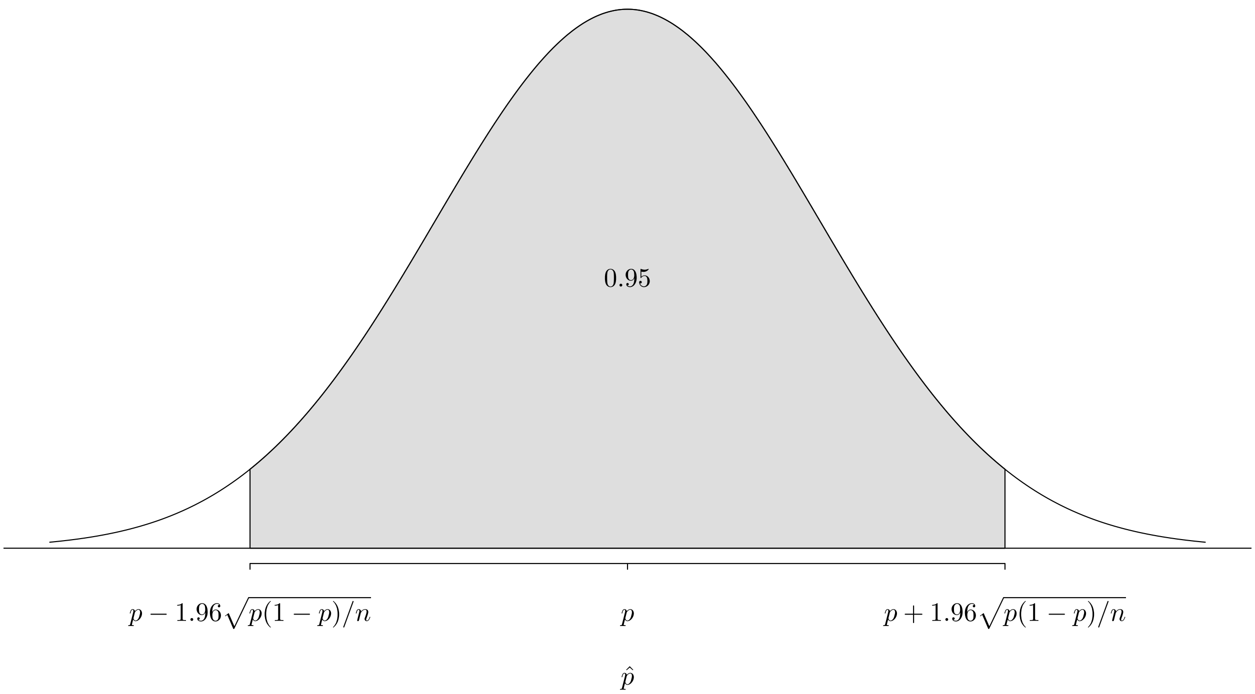
What do we know about the sampling distribution of $\hat{p}$?

1. The mean of $\hat{p}$ is p.
2. The standard deviation (i.e., standard error) of $\hat{p}$ is

$$\sqrt{\frac{p(1-p)}{n}}.$$

3. The shape of the sampling distribution is approximately that of a normal distribution.

This is the sampling distribution of $\hat{p}$.



It is convenient to convert $\hat{p}$ into a z-score using the formula

$$z = \frac{\hat{p} - p}{\sqrt{p(1-p)/n}}.$$

This is the sampling distribution of z.

But we do not know the value of p!

Null and Alternative Hypotheses

Null Hypothesis (H_0): Usually the hypothesis of "no effect" (e.g., nothing is "happening"). In practice the null hypothesis is often that the parameter equals a *specific value* (although we will consider the case when it may be a range of values when we discuss *composite* null hypotheses).

Alternative Hypothesis (H_a): Usually the hypothesis of an "effect" (e.g., something is "happening"). In practice the alternative hypothesis is usually that the parameter is in a *range of values*.

What would the null and alternative hypotheses be for the examples above?

Test Statistics

A **test statistic** measures the discrepancy between the point estimate of the parameter and the hypothesized value of the parameter. A test statistic is computed *under the assumption that the null hypothesis is true.*

Example: The z-score

$$z = \frac{\hat{p} - p}{\sqrt{p(1-p)/n}}$$

is a test statistic. What would be the value of the test statistic for the examples above?

Decision Making

Modus Tollens: If A then B. Not B. Therefore not A.

Example: If someone is a Hobbit (A), then their feet will be hairy (B). Your feet are not hairy (not B). Therefore you are not a Hobbit (not A).

Example: If it rains today (A), then the ground will be wet (B). The ground is not wet (not B). Therefore it did not rain today (not A).

"Probabilistic" Modus Tollens: If H_0 is true (A), then the test statistic *is likely* to be a "typical" value (B). The test statistics is not a "typical" value (not B). Therefore H_0 is *decidedly* false (not A).

Example: If H_0 is true (A), then it is likely that $-1.96 < z < 1.96$ (B). So if $z > 1.96$ or $z < -1.96$ (not B), then we decide that H_0 is not true (not A).

What can we decide?

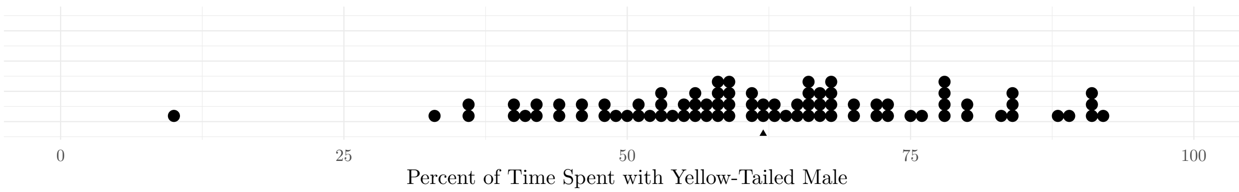
1. The test statistic *is* a "typical" value when H_0 is true. *Do not reject* H_0. The result *is not* statistically significant.
2. The test statistic *is not* a "typical" value when H_0 is true. *Reject* H_0. The result *is* statistically significant.

Note: This is not a true modus tollens argument. This argument can lead us to the wrong conclusion because it is still possible to observe an atypical value of the test statistic even if H_0 is true.

Example: What might we decide for the previous examples?

More Platies!

Do female platies have a preference for a yellow-tailed male?



In 67 out of 84 observations, the female platy spent a majority of her time with the yellow-tailed male. Is this statistically significant?