Monday, Feb 6

Population and Sampling Distributions

A **population distribution** is the probability distribution of *one observation* of a random variable (i.e., x).

A **sampling distribution** is the probability distribution of a *statistic* (e.g., $\bar{x}$ or $\hat{p}$) which is a function of a sample of observations of a random variable (i.e., $x_1, x_2, \dots, x_n$).

The sampling distribution depends on (a) the population distribution and (b) the design.

Properties of the Sampling Distribution of $\bar{x}$

Assume (a) that we have a population distribution of a quantitative variable x with mean μ_x and standard deviation σ_x, and (b) we observe a sample of n observations and compute the mean ($\bar{x}$) from this sample. Note that I will use a subscript on μ and σ to make explicit the variable in question.

Example: Consider the following population distribution, and several sampling distributions of $\bar{x}$ based on samples of $n = 2$, 3, or 4 observations.

Table 1: Population Distribution

x	$P(x)$
20	0.6
30	0.4

$$\mu_x = 24$$
$$\sigma_x \approx 4.9$$

Table 2: Sampling Distribution of $\bar{x}$, $n = 2$

$\bar{x}$	$P(\bar{x})$
20	0.36
25	0.48
30	0.16

$$\mu_{\bar{x}} = 24$$
$$\sigma_{\bar{x}} \approx 3.46$$

Table 3: Sampling Distribution of $\bar{x}$, $n = 3$

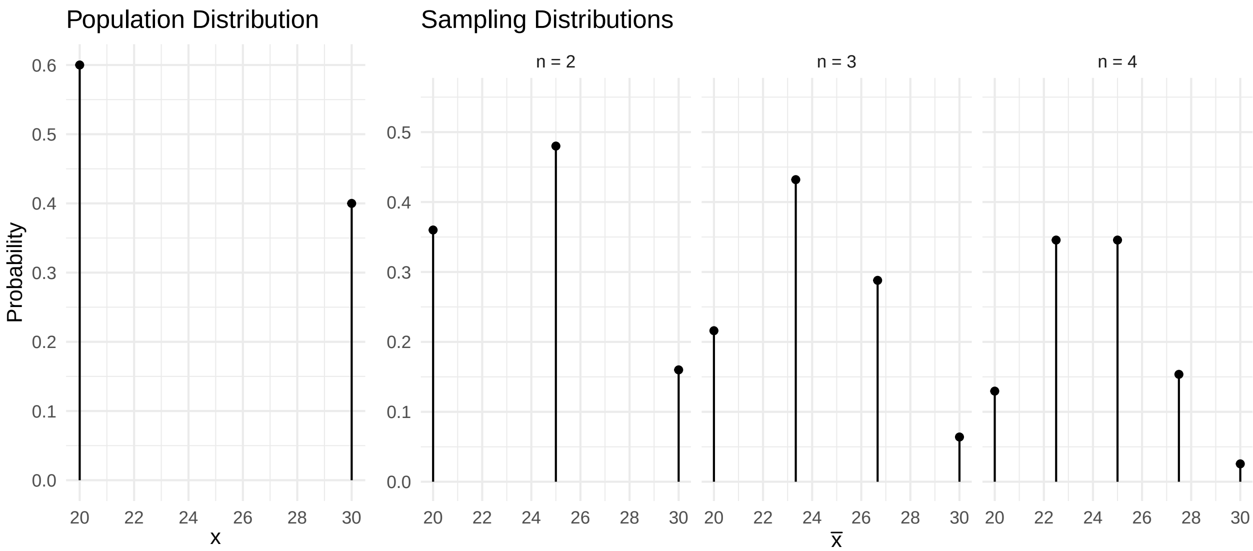
$\bar{x}$	$P(\bar{x})$
20.00	0.216
23.33	0.432
26.67	0.288
30.00	0.064

$$\mu_{\bar{x}} = 24$$
$$\sigma_{\bar{x}} \approx 2.83$$

Table 4: Sampling Distribution of $\bar{x}$, $n = 4$

$\bar{x}$	$P(\bar{x})$
20.0	0.1296
22.5	0.3456
25.0	0.3456
27.5	0.1536
30.0	0.0256

$$\mu_{\bar{x}} = 24$$
$$\sigma_{\bar{x}} \approx 2.45$$

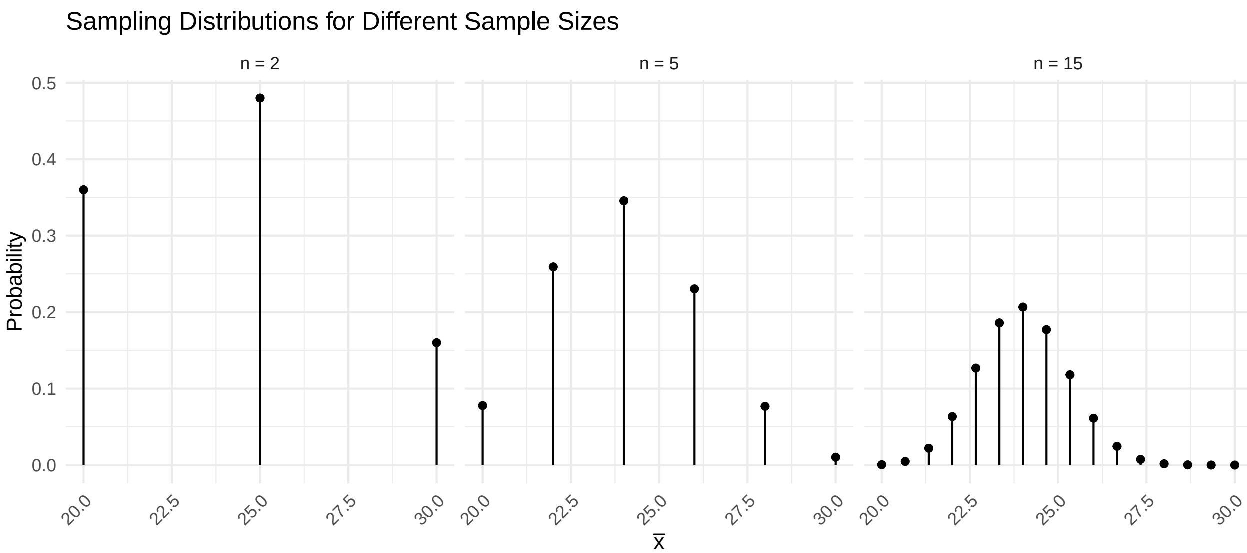

Population Distribution
Sampling Distributions
n = 2
n = 3
n = 4
Probability
x
x̄

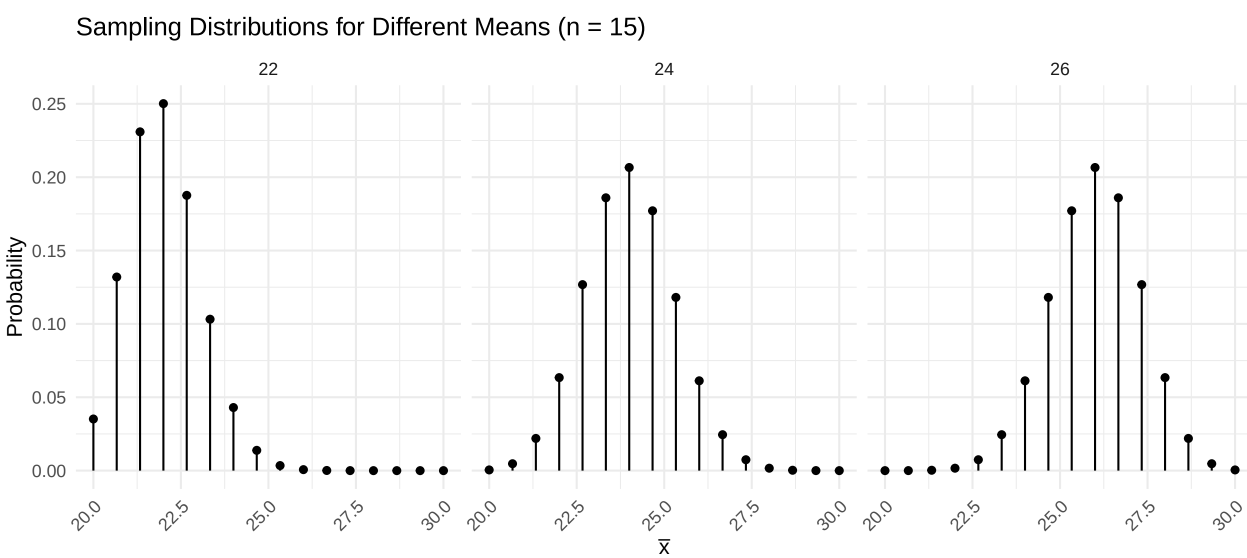
Mean and Standard Deviation of $\bar{x}$

Assume that x has a mean of μ and a standard deviation of σ, and assume a sample of n observations.

1. The *mean* of the $\bar{x}$ is μ_x — i.e., $\mu_{\bar{x}} = \mu_x$.
2. The *standard deviation* of $\bar{x}$ is $\sigma_x/\sqrt{n}$ — i.e., $\sigma_{\bar{x}} = \sigma_x/\sqrt{n}$).

Example: Assuming that $\mu_x = 24$ and $\sigma_x \approx 4.9$, what are the mean and standard deviation of $\bar{x}$ based on a sample of $n = 16$ observations? What about $n = 25$ observations?





Properties of the Sampling Distribution of $\hat{p}$

Assume (a) that we have a population distribution where x has only two values, "success" and "failure," and the probability of a success is p, and assume (b) we observe a sample of n observations and compute the proportion ($\hat{p}$) of observations in the sample that are "successes."

Example: Consider the following population distribution, and several sampling distributions of $\hat{p}$ based on samples of $n = 3$, 4, or 5 observations.

Table 5: Population Distribution

x	$P(x)$
Y	0.7
C	0.3

Note: Here we define Y as a "success" because our proportions will be based on the number of Y's out of n.

Table 6: Sampling Distribution of $\hat{p}$, $n = 3$

$\hat{p}$	$P(\hat{p})$
0	0.027
1/3	0.189
2/3	0.441
1	0.343

$$\mu_{\hat{p}} = 0.7$$
$$\sigma_{\hat{p}} \approx 0.26$$

Table 7: Sampling Distribution of $\hat{p}$, $n = 4$

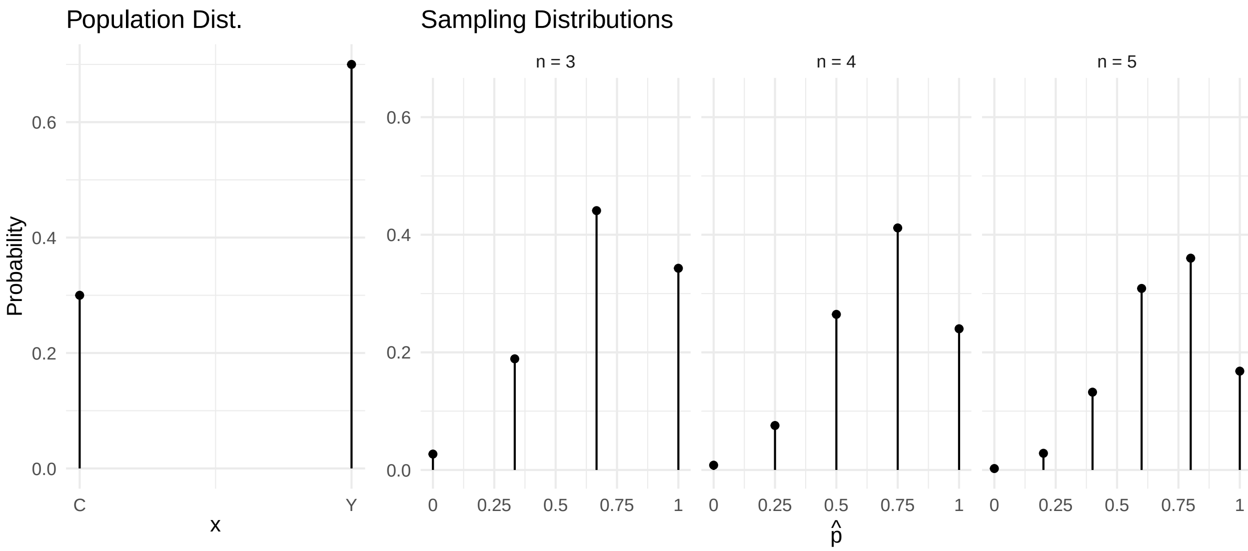
$\hat{p}$	$P(\hat{p})$
0	0.0081
1/4	0.0756
1/2	0.2646
3/4	0.4116
1	0.2401

$$\mu_{\hat{p}} = 0.7$$
$$\sigma_{\hat{p}} \approx 0.23$$

Table 8: Sampling Distribution of $\hat{p}$, $n = 5$

$\hat{p}$	$P(\hat{p})$
0	0.00243
1/5	0.02835
2/5	0.13230
3/5	0.30870
4/5	0.36015
1	0.16807

$$\mu_{\hat{p}} = 0.7$$
$$\sigma_{\hat{p}} \approx 0.2$$



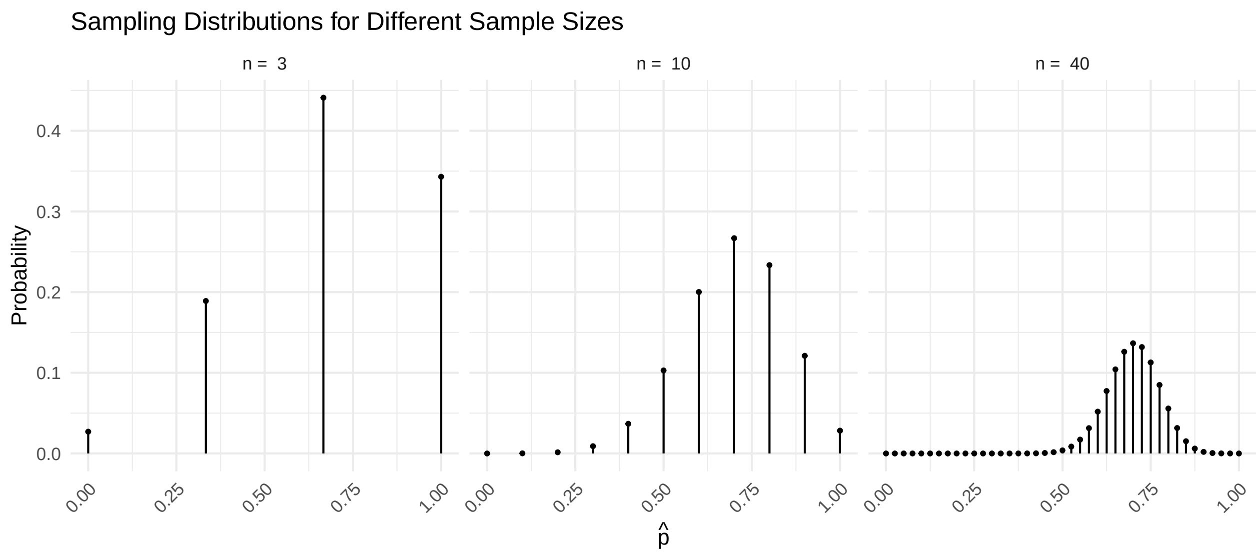
Population Dist.
Sampling Distributions
n = 3
n = 4
n = 5
Probability
0.0
0.2
0.4
0.6
C
Y
x
0
0.25
0.5
0.75
1
p̂

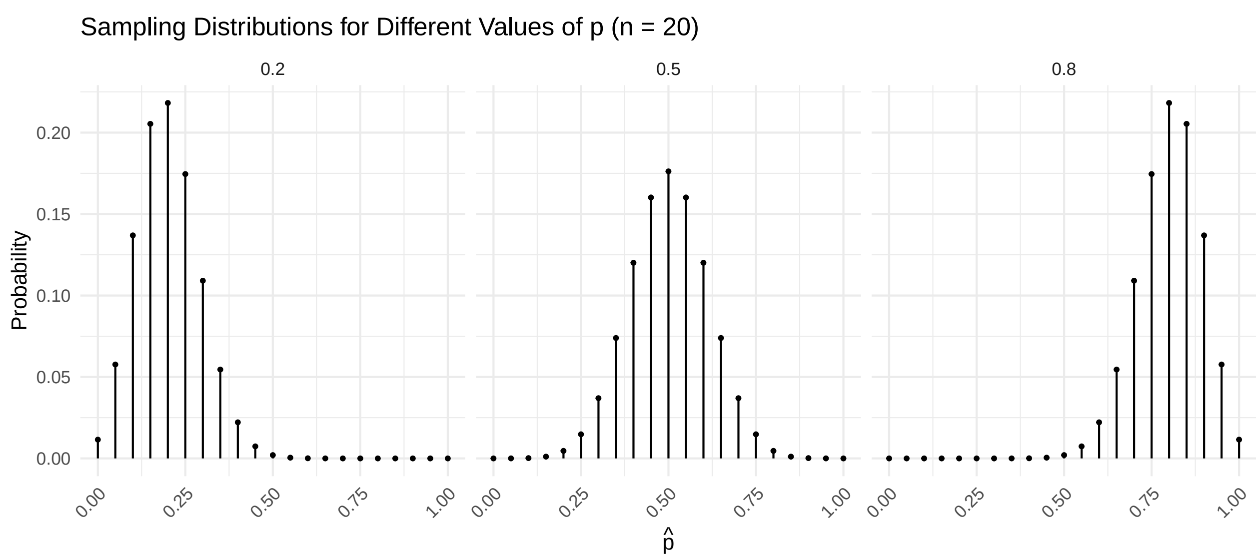
Mean and Standard Deviation of $\hat{p}$

Assume (a) that we have a population distribution where x has only two values, "success" and "failure," and the probability of a success is p, and assume a sample of n observations.

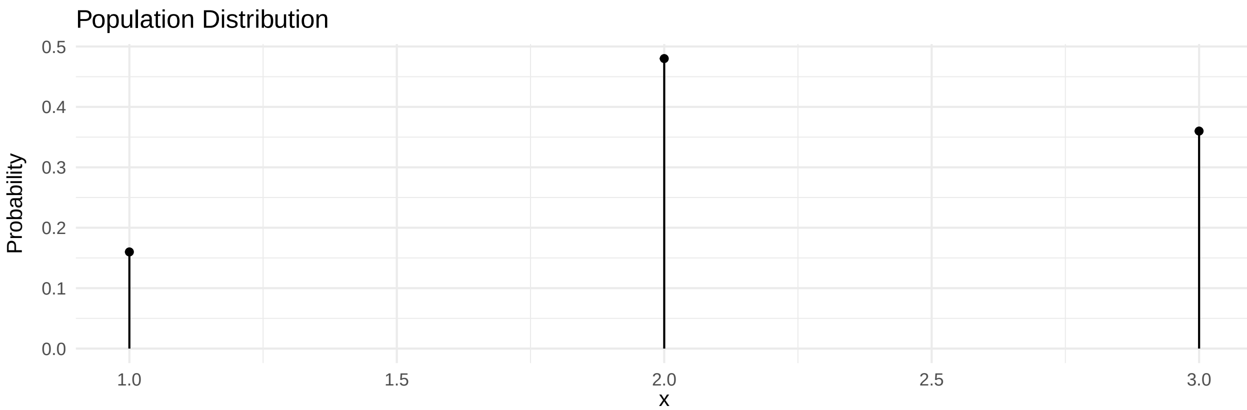
1. The *mean* of $\hat{p}$ is p — i.e., $\mu_{\hat{p}} = p$.
2. The *standard deviation* of $\hat{p}$ is $\sqrt{p(1-p)/n}$ — i.e., $\sigma_{\hat{p}} = \sqrt{p(1-p)/n}$.

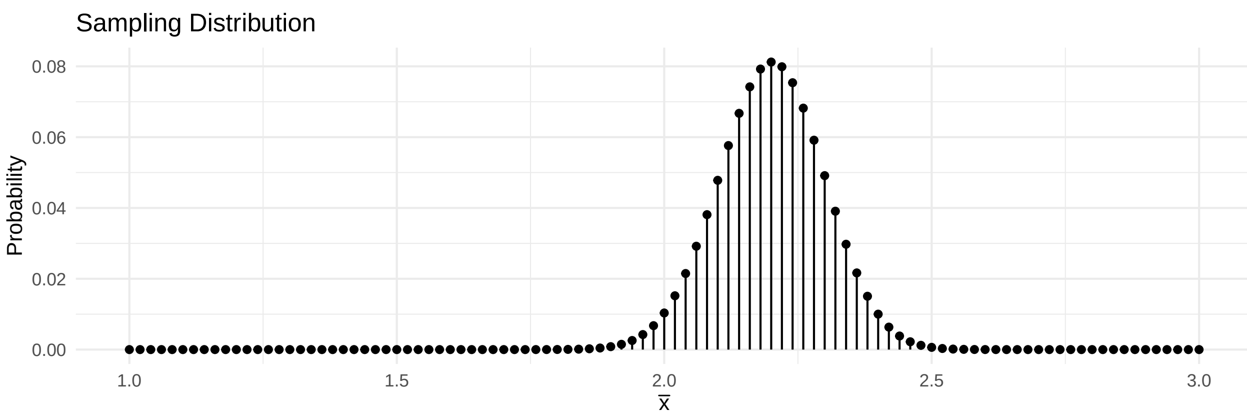
Example: Assuming the population distribution given above with $p = 0.7$, what are the mean and standard deviation of $\hat{p}$ based on a sample of $n = 16$ observations? What about $n = 25$ observations?


Sampling Distributions for Different Sample Sizes
n = 3
n = 10
n = 40
Probability
0.0
0.1
0.2
0.3
0.4
0.00
0.25
0.50
0.75
1.00
p̂


Sampling Distributions for Different Values of p (n = 20)
0.2
0.5
0.8
Probability
0.00
0.05
0.10
0.15
0.20
0.00
0.25
0.50
0.75
1.00
p̂

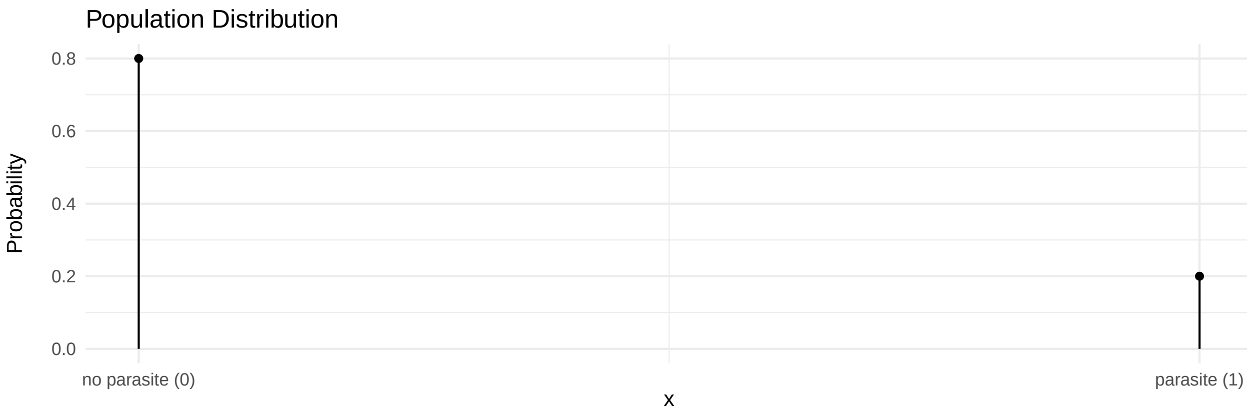
Example: Consider again the trebuchet experiment, but this time with a slightly different population distribution, which is shown below. The mean and standard deviation of x are $\mu_x = 2.2$ and $\sigma_x \approx 0.69$, respectively. A researcher would probably not know μ_x, but could estimate it by firing the trebuchet to create a sample of observations and use $\bar{x}$ to *estimate* μ_x. The sampling distribution of $\bar{x}$ based on a sample of $n =$ 50 observations is also shown below.

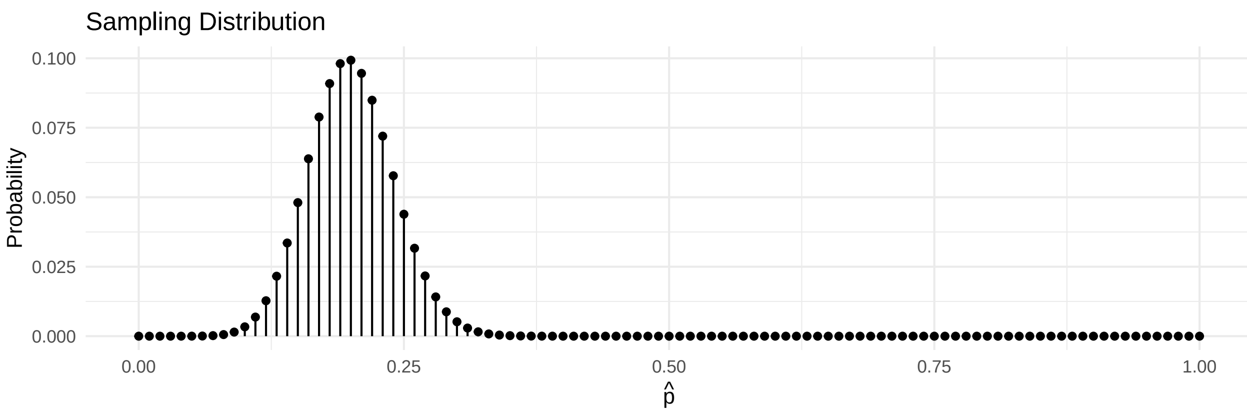




What are the mean and the standard deviation of $\bar{x}$ for such an experiment? Also what is the interval that has approximately a 0.95 probability of containing $\bar{x}$?

Example: Imagine a survey of fish in a lake where 20% of the fish in the lake are infected with a parasite. Let x be whether or not a randomly selected fish has a parasite. The population distribution is shown below. A researcher would probably not know that 20% of the fish in the lake are infected, but could *estimate* the proportion of infected fish in the lake (0.2) using the proportion of infected fish from a sample of observations ($\hat{p}$). The sampling distribution of $\hat{p}$ based on a sample of $n = 100$ observations is also shown below.





What are the mean and standard deviation of $\hat{p}$ from such a survey? Also what is the interval that has approximately a 0.95 probability of containing $\hat{p}$?